Rotational Motion

Question1

The moment of inertia of a solid cylinder of mass 2.5 kg and radius 10 cm about its axis is

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Options:

A.

$0.0725 \mathrm{kgm}^2$

B.

$12500 \mathrm{kgm}^2$

C.

$0.0125 \mathrm{kgm}^2$

D.

$72500 \mathrm{kgm}^2$

Answer: C

Solution:

Moment of inertia of solid cylinder

$$I=\frac{M R^2}{2}=\frac{2.5 \times(0.1)^2}{2}$$
$$=1.25 \times 10^{-2} \mathrm{~kg}-\mathrm{m}^2$$
$$=0.0125 \mathrm{~kg}-\mathrm{m}^2$$

Question2

A circular dise of diameter 0.8 m and mass 4 kg is rolling on a smooth horizontal plane. If 2.56 N m torque is acting on the disc, then its angular acceleration is

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Options:

A.

$8 \mathrm{rads}^{-2}$

B.

$4 \mathrm{rads}^{-2}$

C.

$2 \mathrm{reds}^{-2}$

D.

$16 \mathrm{rads}^{-2}$

Answer: A

Solution:

Step 1: Find the radius and moment of inertia.

The diameter of the disc is 0.8 m, so the radius R is half of that: $R = 0.4$ m.

The mass M is 4 kg. The moment of inertia I for a disc about its center is given by $I = \frac{MR^2}{2}$.

So, $I = \frac{4 \times (0.4)^2}{2}$

$= \frac{4 \times 0.16}{2} = \frac{0.64}{2} = 0.32 \mathrm{~kg} \cdot \mathrm{~m}^2$

Step 2: Use the torque formula to get angular acceleration.

We know the formula: $\tau = I\alpha$, where τ is torque, I is moment of inertia, and α is angular acceleration.

So, $\alpha = \frac{\tau}{I}$.

Plug in the values: $\alpha = \frac{2.56}{0.32} = 8 \mathrm{~rad/s}^2$

Question3

A solid sphere and a solid cylinder have same mass and same radius. The ratio of the moment of inertia of the solid sphere about its diameter and the moment of inertia of the solid cylinder about its axis is

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Options:

A.

3 : 5

B.

4 : 5

C.

3 : 1

D.

2 : 1

Answer: B

Solution:

Let:

- M be the mass of both the solid sphere and the solid cylinder.
- R be the radius of both the solid sphere and the solid cylinder.

1. **Moment of inertia of a solid sphere about its diameter (I_{sphere}):**

   The formula for the moment of inertia of a solid sphere of mass M and radius R about any of its diameters is given by:

   $I_{sphere} = \frac{2}{5}MR^2$

2. **Moment of inertia of a solid cylinder about its axis ($I_{cylinder}$):**

The formula for the moment of inertia of a solid cylinder of mass M and radius R about its central longitudinal axis is given by:

$I_{cylinder} = \frac{1}{2}MR^2$

Now, we need to find the ratio $I_{sphere} : I_{cylinder}$.

Ratio $= \frac{I_{sphere}}{I_{cylinder}} = \frac{\frac{2}{5}MR^2}{\frac{1}{2}MR^2}$

We can cancel out the common terms MR^2 from the numerator and the denominator:

Ratio $= \frac{\frac{2}{5}}{\frac{1}{2}}$

To simplify the fraction, we multiply the numerator by the reciprocal of the denominator:

Ratio $= \frac{2}{5} \times \frac{2}{1}$

Ratio $= \frac{4}{5}$

Therefore, the ratio of the moment of inertia of the solid sphere about its diameter and the moment of inertia of the solid cylinder about its axis is 4 : 5.

Question4

If a solid sphere is rolling without slipping on a horizontal plane, then the ratio of its rotational and total kinetic energies is

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Options:

A.

2 : 5

B.

2 : 7

C.

4 : 3

D.

1 : 2

Answer: B

Solution:

As sphere is in pure rolling, $v = R\omega$

Rotational KE of sphere $= \frac{1}{2} I\omega^2$

$= \frac{1}{2} \cdot \frac{2}{5} mR^2\omega^2 = \frac{1}{5} mv^2$

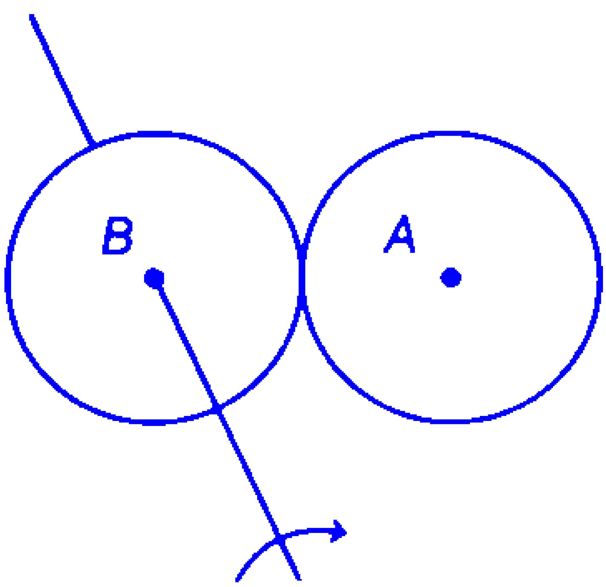
Kinetic energy of translation $= \frac{1}{2} mv^2$

Total KE $= \left(\frac{1}{2} + \frac{1}{5}\right) mv^2 = \frac{7}{10} mv^2$

Required ratio $= \frac{1}{5} \times \frac{10}{7} = \frac{2}{7}$

Question5

As shown in the figure, two thin coplanar circular discs A and B each of mass M' and radius ' r ' are attached to form a rigid body. The moment of inertia of this system about an axis perpendicular to the plane of disc B and passing through its centre is



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Options:

A.

$2Mr^2$

B.

$3Mr^2$

C.

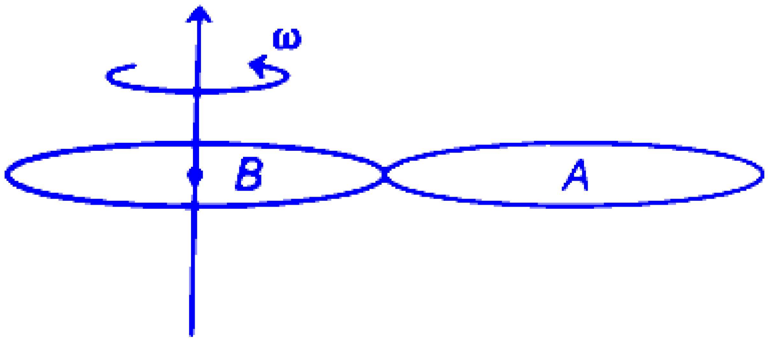
$4Mr^2$

D.

$5Mr^2$

Answer: D

Solution:



Moment of inertia of disc B about given axis,

$I_B = \frac{1}{2}Mr^2$

Moment of inertia of disc A about given axis using parallel axes theorem is,

$I_A = \frac{1}{2}Mr^2 + M \cdot (2r)^2 = \frac{9}{2}Mr^2$

$\therefore M \cdot I$ of system about given axis is,

$I = I_A + I_B = \left(\frac{9}{2} + \frac{1}{2}\right)Mr^2 = 5Mr^2$

Question6

A circular disc of mass 20 kg and radius 1 m is rotating about an axis passing through its centre and perpendicular to its plane with an angular velocity of 2rads^{-1}. Then, the rotational kinetic energy of the disc is

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Options:

A.

100 J

B.

50 J

C.

75 J

D.

20 J

Answer: D

Solution:

Rotational kinetic energy

$$\begin{aligned} K_r &= \frac{1}{2} I \omega^2 \\ &= \frac{1}{2} \times \left(\frac{1}{2} M R^2\right) \omega^2 \\ &= \frac{1}{2} \times \frac{1}{2} \times 20 \times 1^2 \times 2^2 \\ &= 20 \text{ J} \end{aligned}$$

Question7

Radius of gyration of a thin uniform rod of length ' L ' about an axis passing through its centre and perpendicular to its length is

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Options:

A.

$\frac{L}{\sqrt{12}}$

B.

$\frac{L}{12}$

C.

$L\sqrt{12}$

D.

$12L$

Answer: A

Solution:

Given:

A thin uniform **rod** of length L, and we need the **radius of gyration** k about an axis **passing through its centre and perpendicular to its length**.

Formulae:

Moment of inertia (M.I.) about the given axis for a uniform rod is:

$I = \frac{1}{12}ML^2$

The **radius of gyration** k is related to the moment of inertia by

$I = Mk^2$

Substituting:

$Mk^2 = \frac{1}{12}ML^2$

$k^2 = \frac{L^2}{12}$

$k = \frac{L}{\sqrt{12}}$

✅ **Final Answer:**

$\boxed{\text{Option A: } \frac{L}{\sqrt{12}}}$

Question8

A thin circular ring and a circular disc of equal mass are rolling without sliding. If their linear velocities are equal and the total kinetic energy of the disc is 6 J , then the total kinetic energy of the ring is

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Options:

A.

6 J

B.

3 J

C.

8 J

D.

4 J

Answer: C

Solution:

$K_{\text{disc}} \quad = 6 \text{ J}$

$$\therefore \quad \frac{1}{2} I \omega^2 + \frac{1}{2} m v^2 = 6$$
$$\Rightarrow \quad \frac{1}{2} \times \frac{1}{2} m R^2 \cdot \left(\frac{v}{R}\right)^2 + \frac{1}{2} m v^2 = 6$$
$$\Rightarrow \quad \frac{3}{4} m v^2 = 6 \Rightarrow m v^2 = 8 \text{ J} \qquad \ldots (i)$$

$$\text{Thus, } K_{\text{Ring}} = \frac{1}{2} I \omega^2 + \frac{1}{2} m v^2$$
$$= \frac{1}{2} \times (m R)^2 \left(\frac{v}{R}\right)^2 + \frac{1}{2} m v^2$$
$$= \frac{1}{2} m v^2 + \frac{1}{2} m v^2$$
$$= m v^2 = 8 \text{ J}$$

Question9

A solid sphere of mass 4 kg and radius 28 cm is on an inclined plane. If the acceleration of the sphere when it rolls down without sliding is 3.5 ms^{-2}, then the acceleration of the sphere when it slides down without rolling is

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Options:

A.

2.5 ms^{-2}

B.

3.5 ms^{-2}

C.

1.7 ms^{-2}

D.

4.9 ms^{-2}

Answer: D

Solution:

$$a_{\text{rolling}} = \frac{g \sin \theta}{1+\frac{I}{MR^2}} = \frac{g \sin \theta}{1+\frac{\frac{2}{5} MR^2}{MR^2}}$$

$$3.5 = \frac{5}{7}(g \sin \theta)$$

$$\Rightarrow \quad g \sin \theta = 4.9$$

Thus, acceleration of the sphere when it slides down without rolling is $g \sin \theta$

$= 4.9 \text{ m/s}^2$

Question10

A thin uniform circular disc rolls with a constant velocity without slipping on a horizontal surface. Its total kinetic energy is

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Options:

A.

three times its rotational kinetic energy

B.

three times its translational kinetic energy

C.

one and half times its rotational kinetic energy

D.

twice its translational kinetic energy

Answer: A

Solution:

Rotational kinetic energy of disc

$$\begin{aligned} K_r &= \frac{1}{2} I \omega^2 \\ &= \frac{1}{2} \times \frac{1}{2} M R^2 \times \left(\frac{v}{R}\right)^2 \\ &= \frac{1}{4} m v^2 \end{aligned}$$

Total kinetic energy

$$\begin{aligned} &= \frac{1}{2} I \omega^2 + \frac{1}{2} m v^2 \\ &= \frac{1}{2} \times \frac{1}{2} M R^2 \times \left(\frac{v}{R}\right)^2 + \frac{1}{2} m v^2 \\ &= \frac{1}{4} m v^2 + \frac{1}{2} m v^2 \\ &= \frac{3}{4} m v^2 = 3\left(K_r\right) \end{aligned}$$

Question11

Three thin uniform rods each of mass M and length L are placed along the three axes of a cartesian co-ordinate system with one end of all the rods at origin. The moment of inertia of the system of the rods about Z-axis is

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Options:

A.

$\frac{ML^2}{3}$

B.

$\frac{2ML^2}{3}$

C.

$\frac{ML^2}{2}$

D.

ML^2

Answer: B

Solution:

The moment of inertia of a rod about an axis perpendicular to the rod and passing through one end is $\frac{ML^2}{3}$. Two rods are in the X and Y-axis and they both contribute to the moment of inertia about the Z-axis. The rod along

the Z-axis does not contribute to the moment of inertia about the Z-axis. Therefore the total moment of inertia of the system is

$\frac{ML^2}{3} + \frac{ML^2}{3} = \frac{2ML^2}{3}$

Question12

If the radius of gyration of a thin circular ring about an axis passing through its centre and perpendicular to its plane is $10\sqrt{2}$ cm, then its radius of gyration about its diameter is

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Options:

A.

10 cm

B.

20 cm

C.

$10\sqrt{2}$ cm

D.

$20\sqrt{2}$ cm

Answer: A

Solution:

Radius of gyration of ring

$$\begin{aligned} K &= \sqrt{\frac{I}{M}} = \sqrt{\frac{MR^2}{M}} \\ K &= R \end{aligned}$$

$$\Rightarrow 10\sqrt{2} = R$$

$$\Rightarrow \quad R = 10\sqrt{2} \text{ cm}$$

Radius of gyration about diameter

$$K_d = \sqrt{\frac{I_d}{M}} = \sqrt{\frac{\frac{MR^2}{2}}{M}}$$

$$= \frac{R}{\sqrt{2}} = \frac{10\sqrt{2}}{\sqrt{2}} = 10 \text{ cm}$$

Question13

If a wheel starting from rest is rotating with an angular acceleration of $\pi \mathrm{rads}^{-2}$, then the number of rotations made by the wheel in the first 6 seconds time is

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Options:

A.

36

B.

9

C.

18

D.

12

Answer: B

Solution:

$$\begin{aligned} \theta &= \omega_0 t+\frac{1}{2} \alpha t^2 \\ &= 0 \times t+\frac{1}{2} \times \pi \times 6^2 \\ &= 18 \pi \end{aligned}$$

$\therefore$ Number of rotations made by the wheel,

$n=\frac{\theta}{2 \pi}=\frac{18 \pi}{2 \pi}=9$

Question14

The moment of inertia of a solid sphere about its diameter is $20 \mathrm{~kg}-\mathrm{m}^2$. The moment of inertia of a thin spherical shell having the same mass and radius about its diameter is

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Options:

A. $16.6 \mathrm{~kg}-\mathrm{m}^2$

B. $30.3 \mathrm{~kg}-\mathrm{m}^2$

C. $33.3 \mathrm{~kg}-\mathrm{m}^2$

D. $66.6 \mathrm{~kg}-\mathrm{m}^2$

Answer: C

Solution:

Let $I_{\text{solid}}=\frac{2}{5} M R^2$ and $I_{\text{shell}}=\frac{2}{3} M R^2$. Since

$I_{\text{solid}}=20 \Rightarrow \frac{2}{5} M R^2=20 \quad \Longrightarrow \quad M R^2=\frac{5}{2} \times 20=50,$

we get

$I_{\text{shell}}=\frac{2}{3} M R^2=\frac{2}{3} \times 50= 33.3 \mathrm{~kg} \cdot \mathrm{m}^2.$

Answer: Option C (33.3 kg·m²).

Question15

One ring, one solid sphere and one solid cylinder are rolling down on same inclined plane starting from rest The radius of all the three are equal. The object reaches down with maximum velocity is

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Options:

A. solid cylinder

B. solid sphere

C. ring

D. solid sphere and ring

Answer: B

Solution:

For the ring rolling down the inclined plane, the moment of inertia (I),

$I_{\text{ring}} = mr^2$

For solid sphere, $I_{\text{sphere}} = \frac{2}{5}mr^2$ For solid cylinder, $I_{\text{cylinder}} = \frac{1}{2}mr^2$

Since, the solid sphere has the smallest moment of inertia, it will have the greatest translational kinetic energy when it reaches the bottom of the incline, thus solid sphere will reach the bottom of the inclined plane with maximum velocity.

Question16

A uniform metal solid sphere is rotating with angular speed ω_0 about diameter. If the temperature is raised by 50°C. The angular speed will be [given $\alpha_{\text{metal}} = 20 \times 10^{-5\circ}\text{C}^{-1}$]

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Options:

A. $0.95\omega_0$

B. $0.96\omega_0$

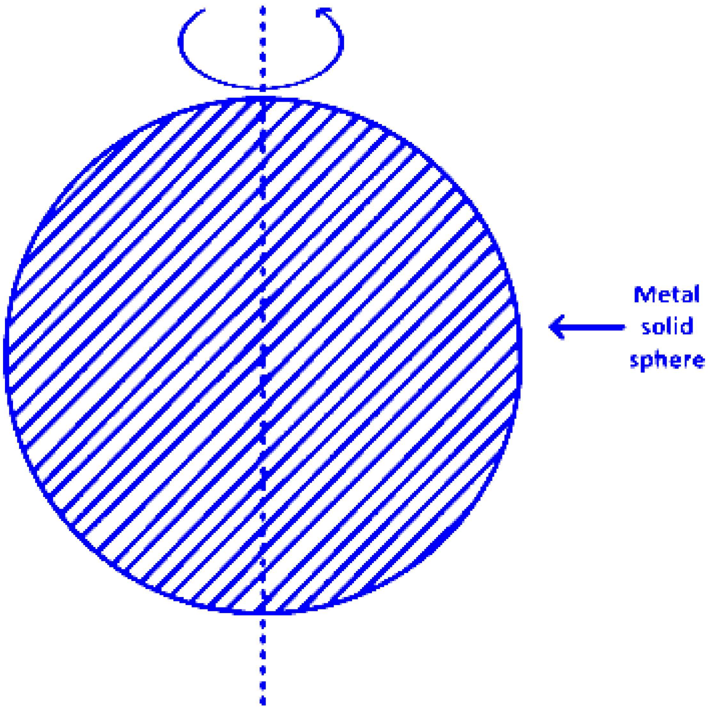
C. $0.98\omega_0$

D. ω_0 (angular velocity is same)

Answer: C

Solution:

Given,



Coeffficient of linear expansion,

$\alpha = 20 \times 10^{-5\circ}\mathrm{C}^{-1}$

Temperature change, $\Delta T = 50^{\circ}\mathrm{C}$

Initial angular speed $= \omega_0$

Using conservation of angular momentum,

$\frac{2}{5} m r^2 \omega_0 = \frac{2}{5} m[r(1+\alpha \Delta T)]^2 \omega'$

$\therefore$ Radius of the sphere after the temperature increase $[r' = r(1+\alpha \Delta T)]$

$$\begin{aligned} \omega' &= \frac{\omega_0}{(1+\alpha \Delta T)^2} = \omega_0[1-2\alpha \Delta t] \\ &= \omega_0\left[1-2 \times 20 \times 10^{-5} \times 50\right] \\ &= \omega_0[1-0.02] = 0.98\omega_0 \end{aligned}$$

The angular speed will be $0.98\omega_0$.

Question17

A ring and a disc of same mass and same diameter are rolling without slipping. Their linear velocities are same, then the ratio of their kinetic energy is

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Options:

A. 0.75

B. 1.33

C. 0.5

D. 2.66

Answer: B

Solution:

Let:

m be the mass

r be the radius

v is the linear velocity

ω is the angular velocity

The condition for rolling without slipping gives us $v = r\omega$.

The total kinetic energy (KE) for any rolling object is the sum of its rotational and translational kinetic energies:

$\mathrm{KE}_{\text{total}} = \mathrm{KE}_{\text{rotational}} + \mathrm{KE}_{\text{translational}}$

The formula for total kinetic energy is:

$\mathrm{KE}_{\text{total}} = \frac{1}{2} I\omega^2 + \frac{1}{2} mv^2$

where I is the moment of inertia.

For the ring:

$I = mr^2$

For the disc:

$I = \frac{mr^2}{2}$

Now, using these values, we can calculate the kinetic energies:

Kinetic Energy of the Disc

$$\mathrm{KE}_{\text{disc}} = \frac{1}{2} \cdot \frac{mr^2}{2} \cdot \left(\frac{v^2}{r^2}\right) + \frac{1}{2} mv^2 = \frac{3}{4} mv^2 \qquad \text{(i)}$$

Kinetic Energy of the Ring

$$\mathrm{KE}_{\text{ring}} = \frac{1}{2} \cdot mr^2 \cdot \left(\frac{v^2}{r^2}\right) + \frac{1}{2} mv^2 = mv^2 \qquad \text{(ii)}$$

Ratio of Kinetic Energy

From equations (i) and (ii):

$$\frac{\mathrm{KE_{ring}}}{\mathrm{KE_{disc}}} = \frac{mv^2}{\frac{3}{4}mv^2} = \frac{4}{3}$$

Therefore, the ratio of the kinetic energy of the ring to the disc is 1.33.

Question18

The moment of inetia of a rod about an axis passing through its centre and perpendicular to its length is $\frac{1}{12}ML^2$, where M is the mass and L is the length of the rod. The rod is bent in the middle, so that the two halves make an angle of 60°. The moment of inertia of the bent rod about the same axis would be

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Options:

A. $\frac{1}{48}ML^2$

B. $\frac{1}{12}ML^2$

C. $\frac{1}{24}ML^2$

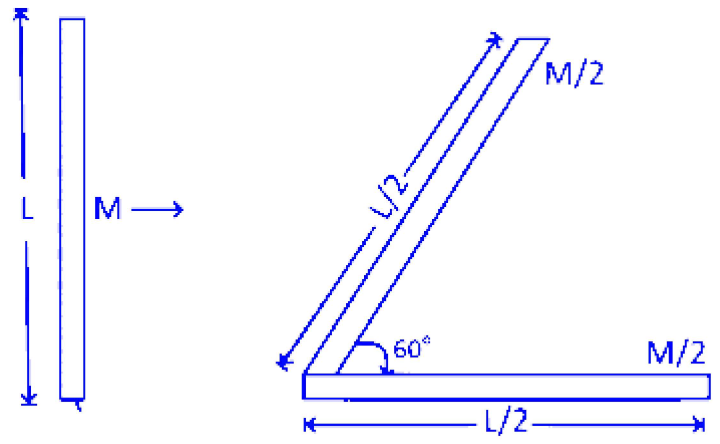
D. $\frac{1}{8\sqrt{3}}ML^2$

Answer: B

Solution:

The total length of a rod is L. So, when the rod is bent at the middle, then each part have same length $\left(\frac{L}{2}\right)$ and mass $\left(\frac{M}{2}\right)$.

Moment of inertia of each part about axis passing through its one end.



$$= \frac{1}{3}\left(\frac{M}{L}\right)\left(\frac{1}{2}\right)^2$$

Thus, Net moment of inertia

$$I = \frac{1}{3}\left[\frac{M}{2}\right]\left[\frac{L}{2}\right]^2 + \frac{1}{3}\left[\frac{M}{2}\right]\left[\frac{L}{2}\right]^2$$

$$= \frac{1}{3}\frac{ML^2}{8} + \frac{1}{3}\frac{ML^2}{8}$$

$$\Rightarrow I = \frac{ML^2}{12}$$

Question19

A uniform rod of length $2L$ is placed with one end in contact with the earth and is then inclined at an angle α to the horizontal and allowed to fall without slipping at contact point. When it becomes horizontal, its angular velocity will be

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Options:

A. $\sqrt{\frac{3g\sin\alpha}{2L}}$

B. $\sqrt{\frac{2L}{3g\sin\alpha}}$

C. $\sqrt{\frac{6g\sin\alpha}{L}}$

D. $\sqrt{\frac{L}{g\sin\alpha}}$

Answer: A

Solution:

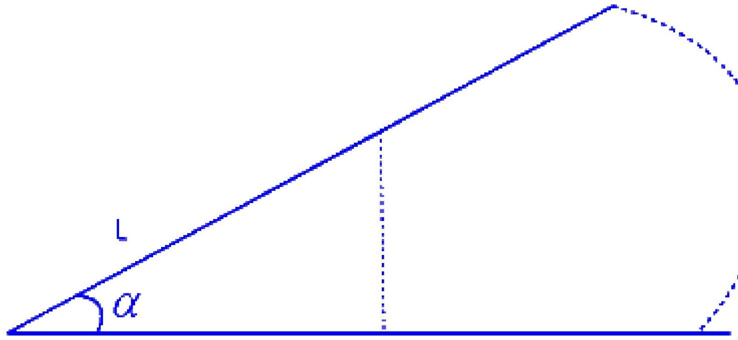
By the conservation of energy

Loss in potential energy of rod = Gain of rotational kinetic energy

$$mg \times \frac{2L}{2} \sin\alpha = \frac{1}{2} I\omega^2$$

$$mg \times \frac{2L}{2} \sin\alpha = \frac{1}{2} \times \frac{m(2L)^2}{3} \times \omega^2$$

Here, $\omega = \sqrt{\frac{3g\sin\alpha}{2L}}$



Question20

A solid cylinder rolls down on an inclined plane of height h and inclination θ. The speed of the cylinder at the bottom is

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Options:

A. $\sqrt{\frac{gh}{2}}$

B. $\sqrt{\frac{3gh}{2}}$

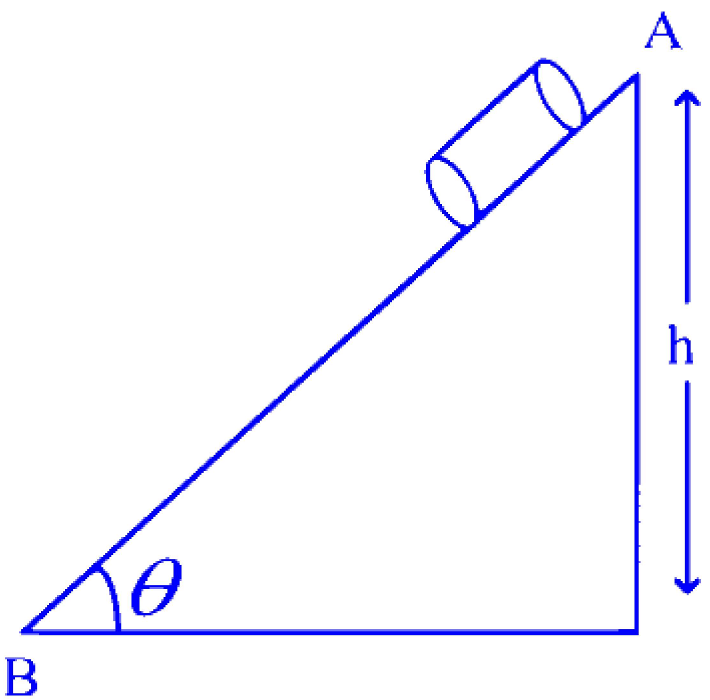
C. $\sqrt{2gh}$

D. $\sqrt{\frac{4gh}{3}}$

Answer: D

Solution:

According to work energy theorem,



Total energy at point A = Total energy at point B

$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 \qquad \ldots(i)$

(Rotational energy)

For cylinder, $I = \frac{1}{2}MR^2$ and

Linear velocity, $v = R\omega \Rightarrow \omega = \frac{v}{R}$

Substitute it in Eq. (i)

$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2} \times \frac{1}{2} \times MR^2 \times \frac{v^2}{R^2}$$
$$\Rightarrow \quad gh = \frac{3}{4}v^2 \Rightarrow v = \sqrt{\frac{4}{3}gh}$$

Question21

Three particles of each mass m are kept at the three vertices of an equilateral triangle of side l. The moment of inertia of a system of the particles about any side of the triangle is

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Options:

A. $\frac{ml^2}{4}$

B. ml^2

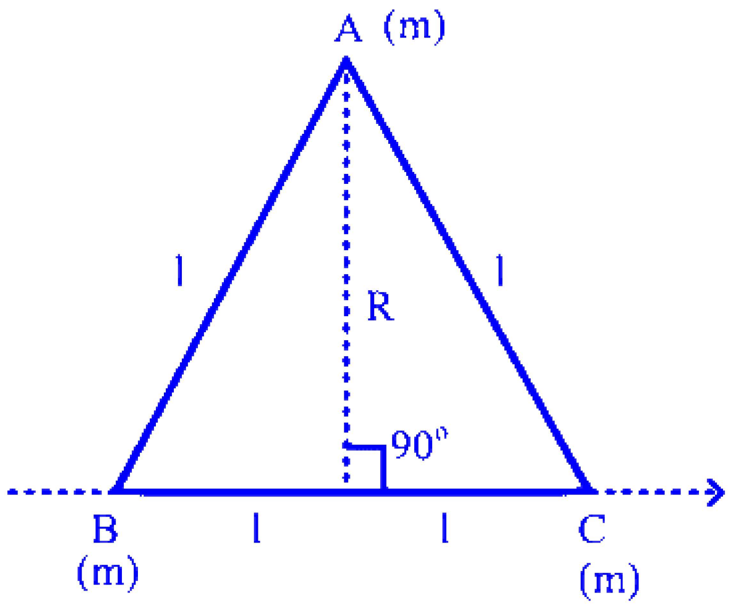
C. $\frac{3}{4}ml^2$

D. $\frac{2}{3}ml^2$

Answer: C

Solution:

Let the axis of rotation be BC.



So, moment of inertia is given by,

$$I = I_A + I_B + I_C$$
$$= m_A R^2 + m_B(0) + m_C(0)$$

As both masses lie on axis of rotation, separation distance is zero.

$$I = m_A R^2 \qquad \ldots (i)$$

Using Pythagoras theorem,

$$l^2 = \frac{l^2}{4} + R^2 \Rightarrow R^2 = \frac{3}{4}l^2$$
$$I = m_A \frac{3}{4} l^2 \quad \text{[From Eq. (i)]}$$
$$\Rightarrow \quad I = \frac{3}{4}ml^2 \quad (\text{ as } m_A = m)$$

Question22

The masses of a solid cylinder and hollow cylinder ate 3.2 kg and 1.6 kg respectively. Both the solid and hollow cylinders start from rest from the top of an inclined plane and roll down without slipping. If both the cylinder have equal radius and the acceleration of solid cylinder is $4 \mathrm{~ms}^{-1}$, the acceleration of hollow cylinder is

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Options:

A. $2 \mathrm{~ms}^{-2}$

B. $9 \mathrm{~ms}^{-2}$

C. $6 \mathrm{~ms}^{-2}$

D. $3 \mathrm{~ms}^{-2}$

Answer: D

Solution:

Given:

Mass of solid cylinder, $m_s = 3.2 \mathrm{~kg}$

Mass of hollow cylinder, $m_h = 1.6 \mathrm{~kg}$

Acceleration of the solid cylinder, $a_s = 4 \mathrm{~m/s}^2$

Moment of Inertia

For a solid cylinder:

$I_s = \frac{1}{2} m R^2$

For a hollow cylinder:

$I_h = m R^2$

Equation of Motion for Rolling Objects

The equation for motion is given by:

$mg\sin\theta = ma + I\frac{a}{R^2}$

$g\sin\theta = a\left(1 + \frac{I}{mR^2}\right)$

Calculations

Solid Cylinder

For the solid cylinder:

$g\sin\theta = 4\left(1 + \frac{mR^2}{2mR^2}\right)$

$g\sin\theta = 6$

Hollow Cylinder

For the hollow cylinder:

$g\sin\theta = a_h\left(1 + \frac{mR^2}{mR^2}\right)$

$6 = a_h(1 + 1)$

$a_h = 3\,\mathrm{m/s}^2$

Therefore, the acceleration of the hollow cylinder is $3\,\mathrm{m/s}^2$.

Question23

A solid sphere of mass 50 kg and radius 20 cm is rotating about its diameter with an angular velocity d 420 rpm . The angular momentum of the sphere is

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Options:

A. $8.8\ \mathrm{J-s}$

B. $70.4\ \mathrm{J-s}$

C. $17.6\ \mathrm{J-s.}$

D. $35.2\ \mathrm{J-s}$

Answer: D

Solution:

Given:

Mass of the sphere, $m = 50\,\text{kg}$

Radius of the sphere, $r = 20\,\text{cm} = 0.2\,\text{m}$

Angular velocity, $\omega = 420 \times \frac{2\pi}{60} = 14\pi\,\text{rad/s}$

The moment of inertia I of a solid sphere rotating about its diameter is given by:

$I = \frac{2}{5}mr^2$

Calculating the moment of inertia:

$$
\begin{aligned}
I &= \frac{2}{5} \times 50 \times (0.2)^2 \\
&= \frac{2}{5} \times 50 \times 0.04 \\
&= 20 \times 0.04 \\
&= 0.8\,\text{kg} \cdot \text{m}^2
\end{aligned}
$$

The angular momentum L of the rotating sphere is calculated as:

$L = I \cdot \omega$

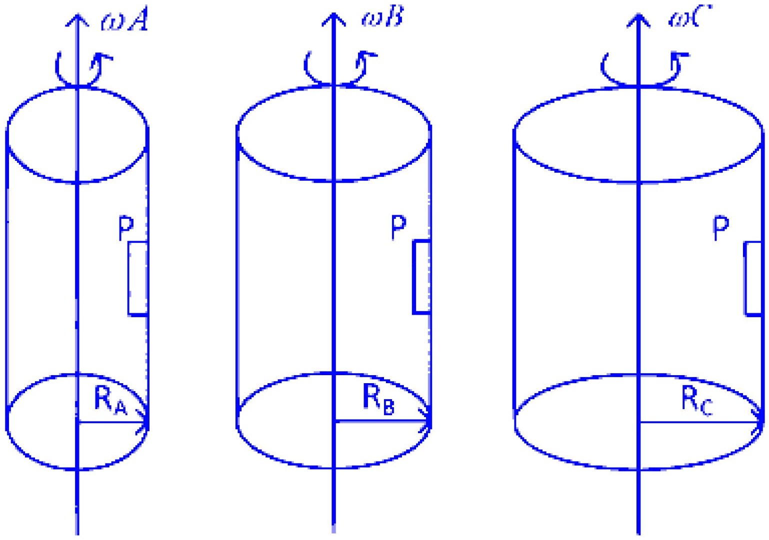
Substitute the known values:

$$
\begin{aligned}
L &= 0.8 \times 14\pi \\
&= 0.8 \times 14 \times \frac{22}{7} \\
&= 44 \times 0.8 \\
&= 35.2\,\text{kg} \cdot \text{m}^2/\text{s} \\
&= 35.2\,\text{J} \cdot \text{s}
\end{aligned}
$$

Therefore, the angular momentum of the sphere is $35.2\,\text{J} \cdot \text{s}$.

Question24

A block P is rotating in contact with the vertical wall of a rotor as shown in figures A, B, C. The relation between angular velocities ω_A, ω_B and ω_C, so that block does not slide down. ($R_A < R_b < R_c$ radii)



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Options:

A. $\omega_A < \omega_B < \omega_C$

B. $\omega_A = \omega_B = \omega_C$

C. $\omega_C < \omega_B < \omega_A$

D. $\omega_C = \omega_A + \omega_B$

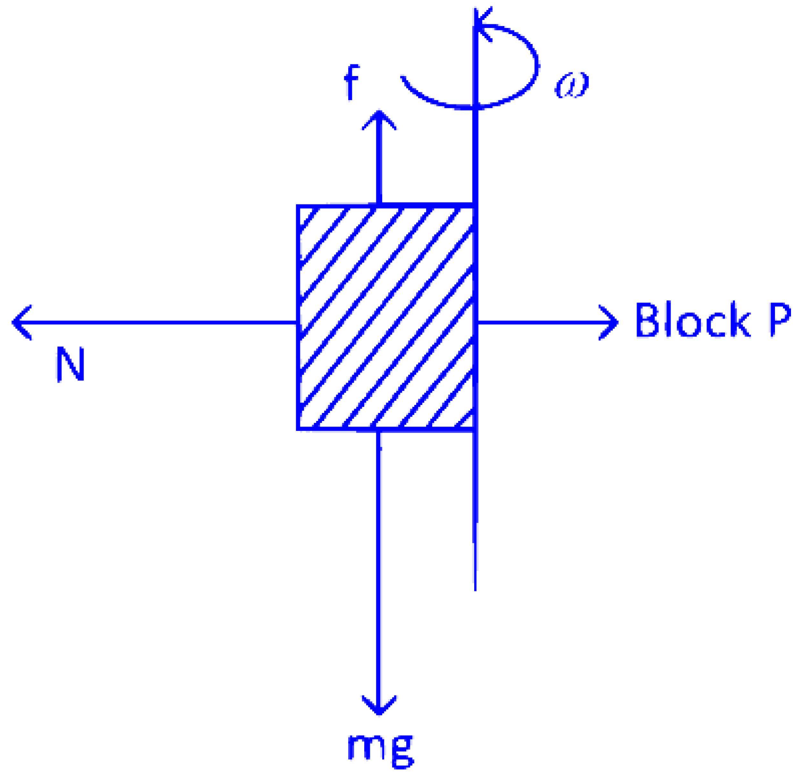
Answer: C

Solution:

Given, $\boldsymbol{R}_A < \boldsymbol{R}_B < \boldsymbol{R}_C$

Frictional force opposes the body from sliding

Here, $f = mg \quad \ldots$ (i)



Normal reaction provided necessary centripetal force.

$N = m\omega^2 R \quad$ (where R is radius)

$f = \mu N = \mu m\omega^2 R \quad \ldots$ (ii)

Substitute the value of f in Eq. (i),

$$mg = \mu m\omega^2 R$$
$$\omega = \frac{\sqrt{g}}{\mu R} \quad \ldots \text{(iii)}$$

From here, $\omega \propto \frac{1}{R}$

$\therefore$ The correct sequence of angular velocity is

$\omega_C < \omega_B < \omega_A$

Question25

The moment of inertia of a solid sphere of mass 20 kg and diameter 20 cm about the tangent to the sphere is

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Options:

A. $0.24 \mathrm{kgm}^2$

B. $0.14 \mathrm{kgm}^2$

C. $0.28 \mathrm{kgm}^2$

D. $0.08 \mathrm{kgm}^2$

Answer: C

Solution:

To find the moment of inertia of a solid sphere with a mass of 20 kg and a diameter of 20 cm about a tangent to the sphere, we follow these steps:

First, find the moment of inertia of the sphere about its center of mass. The formula for the moment of inertia (MI) of a solid sphere about its center is given by:

$MI = \frac{2}{5} MR^2$

where:

M is the mass of the sphere (20 kg),

R is the radius of the sphere.

Given the diameter is 20 cm, the radius R is:

$R = \frac{20}{2}\,\mathrm{cm} = 10\,\mathrm{cm} = 0.10\,\mathrm{m}$

Substituting the values into the formula:

$MI = \frac{2}{5} \times 20 \times (0.10)^2 = 0.08\,\mathrm{kg\,m}^2$

Next, we use the theorem of parallel axes. This theorem states that the moment of inertia of a body about any axis parallel to an axis through its center of mass is the sum of the moment of inertia about the center of mass and the product of the mass and the square of the distance between the two axes.

For a tangent to the sphere, the distance is R (which is the radius), so:

$\text{MI about tangent} = \frac{2}{5} MR^2 + MR^2 = \frac{7}{5} MR^2$

Substitute the values:

$= \frac{7}{5} \times 20 \times (0.10)^2 = 0.28\,\mathrm{kg\,m}^2$

Therefore, the moment of inertia of the solid sphere about a tangent is $0.28\,\mathrm{kg\,m}^2$.

Question26

A solid cylinder rolls down an inclined plane without slipping. If the translation kinetic energy of the cylinder is 140 J . The total kinetic

energy of the cylinder is

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Options:

A. 105 J

B. 70 J

C. 210 J

D. 280 J

Answer: C

Solution:

The problem discusses a solid cylinder rolling down an inclined plane without slipping. Here's a step-by-step breakdown to find the total kinetic energy of the cylinder:

Translation Kinetic Energy

The translation kinetic energy (K_{trans}) is given by:

$K_{\text{trans}} = \frac{1}{2}mv^2$

Given that:

$K_{\text{trans}} = 140 \text{ J}$

We know the relationship between linear velocity (v) and angular velocity (ω) for rolling without slipping is:

$v = \omega R$

This gives:

$140 = \frac{1}{2}m(\omega R)^2$

Rearranging, we find:

$280 = mR^2\omega^2 \ldots \text{(i)}$

Rotational Kinetic Energy

The rotational kinetic energy (K_{rot}) can be expressed as:

$K_{\text{rot}} = \frac{1}{2}I\omega^2$

For a solid cylinder, the moment of inertia (I) is:

$I = \frac{mR^2}{2}$

Substitute this into the expression for rotational kinetic energy:

$K_{\text{rot}} = \frac{1}{2} \times \frac{mR^2}{2} \times \omega^2$

$K_{\text{rot}} = \frac{280}{4}$ [Using Eq. (i)]

$K_{\text{rot}} = 70 \text{ J}$

Total Kinetic Energy

The total kinetic energy of the cylinder is the sum of its translation and rotational kinetic energies:

$K = K_{\text{trans}} + K_{\text{rot}}$

Thus:

$K = 140 + 70 = 210 \text{ J}$

Therefore, the total kinetic energy of the cylinder is 210 J.

Question27

The moment of inertia of a solid cylinder and a hollow cylinder of same mass and same radius about the axes of the cylinders are I_1 and I_2. The relation between I_1 and I_2 is

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Options:

A. $L_1 < I_2$

B. $t_1 = l_2$

C. $l_1 > I_2$

D. $L_1 = I_2 = 0$

Answer: A

Solution:

Given:

Mass of both cylinders = M

Radius of both cylinders = R

The moment of inertia for a solid cylinder and a hollow cylinder about their axes are as follows:

For the solid cylinder:

$I_1 = \frac{MR^2}{2}$

For the hollow cylinder:

$I_2 = MR^2$

From these formulas, it is clear that:

$I_1 < I_2$

Question28

A solid cylinder of radius R is at rest at a height h on an inclined plane. If it rolls down then its velocity on reaching the ground is

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Options:

A. $\sqrt{\frac{5gh}{3}}$

B. $\sqrt{\frac{2h}{3g}}$

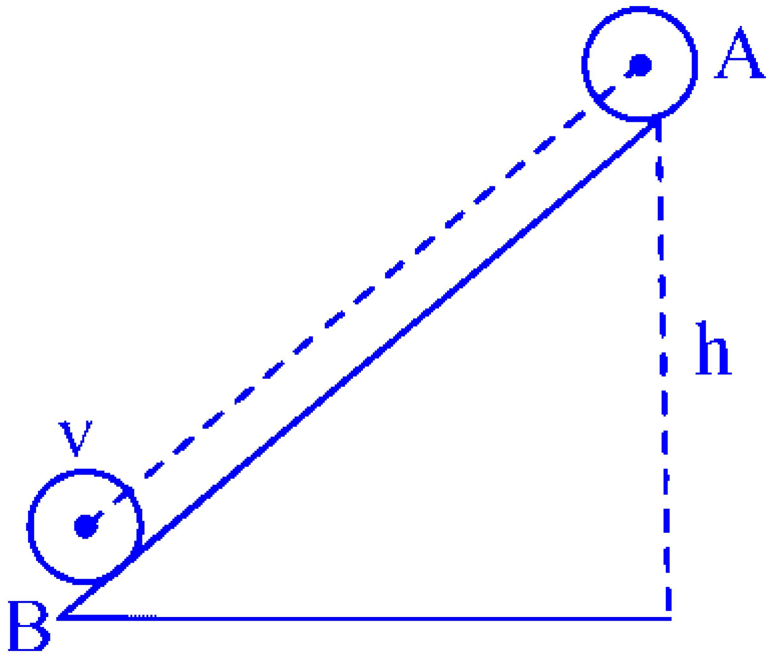
C. $\sqrt{\frac{2gh}{3}}$

D. $\sqrt{\frac{4gh}{3}}$

Answer: D

Solution:

A solid cylinder with radius R is at rest at a height h on an inclined plane. As it rolls down and reaches the ground, its velocity can be determined as follows:



Let v be the velocity of the solid cylinder when it reaches the ground.

By applying the conservation of energy principle:

Total energy at point A = Total energy at point B

At point A, the total energy is: $mgh + 0$ (Potential Energy at height h)

At point B, the total energy is: $\frac{1}{2}I\omega^2 + \frac{1}{2}mv^2 + 0$ (Rotational and Kinetic Energy at ground level)

Substituting the moment of inertia I for a solid cylinder ($I = \frac{1}{2}MR^2$) and the angular velocity ($\omega = \frac{v}{R}$):

$$mgh = \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v}{R}\right)^2 + \frac{1}{2}mv^2$$

$$gh = \frac{v^2}{4} + \frac{v^2}{2}$$

$$gh = \frac{3v^2}{4}$$

$$v^2 = \frac{4gh}{3}$$

$$v = \sqrt{\frac{4gh}{3}}$$

Question29

A solid sphere of radius R has its outer half removed, so that its radius becomes $(R/2)$. Then its moment of inertia about the diameter is

Options:

A. becomes $\frac{1}{2}$ of its initial value.

B. is unchanged.

C. becomes $\frac{1}{16}$ of initial value.

D. becomes $\frac{1}{32}$ of initial value.

Answer: D

Solution:

Let m be the mass of the solid sphere of radius R, then moment of inertia about its diameter

$I = \frac{2}{5} M R^2$

$$\begin{aligned} \text{Here, } m &= \text{ volume } \times \text{ density} \\ &= \frac{4}{3}\pi R^3 \rho \\ \because \quad I &= \frac{2}{5} \times \frac{4}{3}\pi R^3 \rho \cdot R^2 \\ I &= \frac{8}{15}\pi R^5 \rho \quad \text{... (i)} \end{aligned}$$

where, ρ is density of sphere.

When outer half of sphere is removed, then its new volume,

$$\begin{aligned} V' &= \frac{4}{3}\pi\left(\frac{R}{2}\right)^3 \\ &= \frac{4}{3}\pi\frac{R^3}{8} = \frac{\pi R^3}{6} \end{aligned}$$

$\because$ New mass of remaining part of sphere,

$M' = V' \times \rho = \frac{\pi R^3 \rho}{6}$

$\because$ Moment of inertia of newly formed sphere,

$$\begin{aligned} I' &= \frac{2}{5}M'\left(\frac{R}{2}\right)^2 \\ &= \frac{2}{5}\frac{\pi R^3 \rho}{6} \times \frac{R^2}{4} = \frac{\pi R^5 \rho}{60} \\ \Rightarrow \quad I' &= \frac{1}{32}\left[\frac{8}{15}\pi R^5 \rho\right] = \frac{1}{32} \times I \quad \text{[From Eq. (i)]} \\ \Rightarrow \quad I' &= \frac{I}{32} \end{aligned}$$

Question30

Consider a disc of radius R and mass M. A hole of radius $\frac{R}{3}$ is created in the disc, such that the centre of the hole is $\frac{R}{3}$ away from centre of the disc. The moment of inertia of the system along the axis perpendicular to the disc passing through the centre of the disc is

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Options:

A. $\frac{MR^2}{2}$

B. $\frac{13}{27}MR^2$

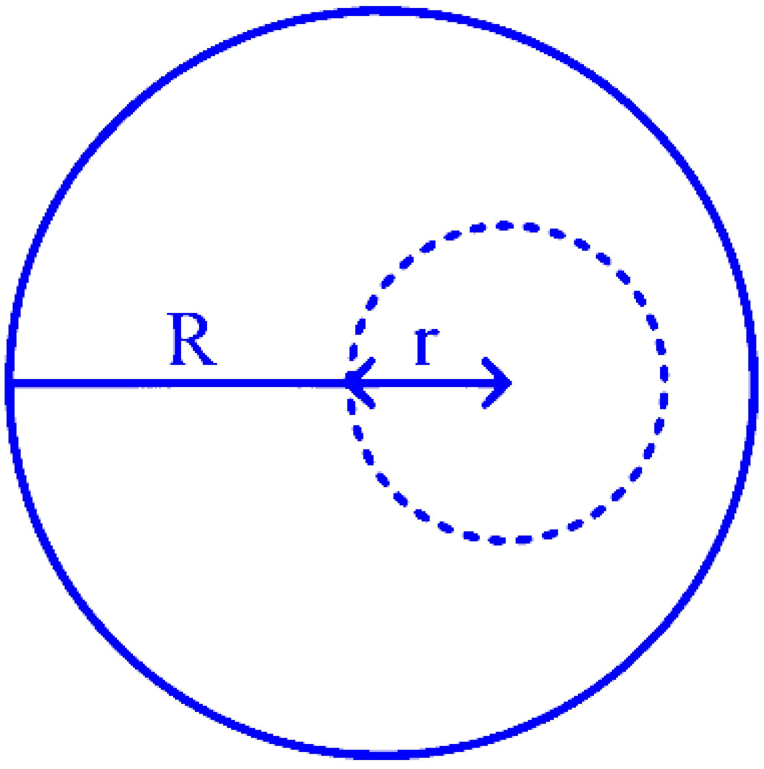
C. $\frac{1}{3}MR^2$

D. $4MR^2$

Answer: B

Solution:

According to question, a disc of $r=\frac{R}{3}$ is removed from the bigger disc of radius R.



The mass of reduced disc is $\frac{M}{9}$ because mass is directly proportional to the area for uniform disc.

Moment of inertia of total disc, $I_{\text{total}} \quad = \frac{MR^2}{2}$

Moment of inertia of removed disc $I_{\text{removed}} \quad = I_{\text{CM}} + \frac{M}{9} \times \left(\frac{R}{3}\right)^2$

$$
\begin{aligned}
I_{\text{removed}} \quad &= \frac{M}{9} \frac{\left(\frac{R}{3}\right)^2}{2} + \frac{MR^2}{81} \\
&= \frac{MR^2}{81}\left(\frac{1}{2} + 1\right) = \frac{MR^2}{81} \times \frac{3}{2} = \frac{MR^2}{54}
\end{aligned}
$$

The moment of inertia of remaining disc $=$

$$
\begin{aligned}
& I_{\text{total}} \quad - I_{\text{removed}} \\
& = \frac{MR^2}{2} - \frac{MR^2}{54} = \frac{MR^2(27-1)}{54} \\
& = \frac{26}{54} MR^2 = \frac{13}{27} MR^2
\end{aligned}
$$

Question31

As solid sphere of mass M and radius R spins about an axis passing through its centre making 600 rpm. Its kinetic energy of rotation is

Options:

A. $\frac{2\pi^2}{5}MR$

B. $\frac{2\pi}{5}M^2R^2$

C. 80πMR

D. $80\pi^2MR^2$

Answer: D

Solution:

Given, mass of solid sphere $= M$

Radius of solid sphere $= R$

Angular frequency, $\omega = 600\text{rpm} = 20\pi$ rad/s

As we know that,

Kinetic energy, $E = \frac{1}{2}I\omega^2$

$$= \frac{1}{2} \times \frac{2}{5}MR^2(20\pi)^2$$
$$= \frac{MR^2}{5}(400\pi^2) = 80\pi^2MR^2$$

Question32

Two fly wheels A and B are mounted side by side with frictionless bearings on a common shaft. Their moments of inertia about the shaft are $5.0\ \text{kg} - \text{m}^2$ and $20.0\ \text{kg} - \text{m}^2$, respectively. Wheel A is made to rotate at 10 rev per second. Wheel B, initially stationary, is now coupled to A with the help of a clutch. The rotation speed of the wheels will become

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Options:

A. $2\sqrt{5}$ rps

B. 0.5 rps

C. 2 rps

D. 3 rps

Answer: C

Solution:

We know the moment of inertia is a measure of how hard it is to spin an object. For wheel A, it is $5\ \mathrm{kg}-\mathrm{m}^2$. For wheel B, it is $20\ \mathrm{kg}-\mathrm{m}^2$.

Wheel A is spinning at 10 revolutions per second (10 rps), and wheel B is not moving at all at the start (0 rps).

Let's call the speed that both wheels spin at after they are connected ω.

When the wheels are joined, the total rotational momentum (angular momentum) stays the same, because there is no friction or any loss.

The total rotational momentum before the clutch is connected is:
$I_1\omega_1 + I_2\omega_2$

After they are joined, the total moment of inertia is $I_1 + I_2$ and both wheels move at the new speed ω:

$I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega$

Now substitute the numbers:
$5 \times 10 + 20 \times 0 = (5 + 20)\omega$

$50 = 25\omega$

$\omega = \frac{50}{25} = 2$ rps

This means after the clutch is connected, both wheels will rotate together at 2 rps.

Question33

A sphere and a hollow cylinder withoutslipping, roll down two separate inclinedplanes A and B, respectively. They cover samedistance in a given duration. If the angle ofinclination of plane A is 30°, then the angleof inclination of plane B must be(approximately)

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Options:

A. 60°

B. 53°

C. 45°

D. 37°

Answer: C

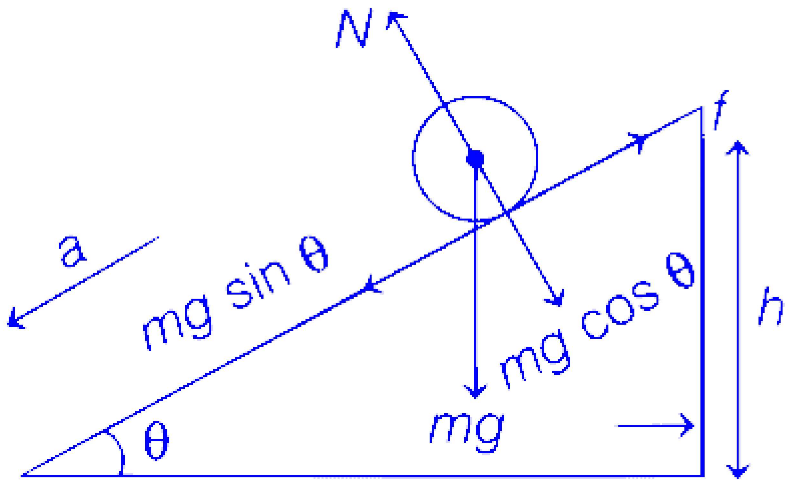
Solution:

Moment of inertia of sphere and hollow cylinder are,

$I_{sphere} = \frac{2}{5} M R^2$

and $I_{cylinder} = M R^2$

Angle of inclination, $\theta = 30\Upsilon$



Let, acceleration of body = a

Friction force = f

Mass of body = m

Radius of body = R

$mg \sin\theta - f = ma$ ..... (i)

Torque, $\tau = Rf = I\alpha$

$\Rightarrow f = I\alpha / R$ ..... (ii)

From Eqs. (i) and (ii), we get

$mg \sin \theta - \frac{I\alpha}{R} = ma$

Angular acceleration, $\alpha = a/R$

$$\Rightarrow \quad mg \sin \theta - \frac{Ia}{R^2} = ma$$
$$\Rightarrow \quad mg \sin \theta = a\left(m + \frac{I}{R^2}\right)$$
$$\Rightarrow \quad a = \frac{mg \sin \theta}{m + \frac{I}{R^2}} = \frac{g \sin \theta}{1 + \frac{I}{mR}}$$

$\because$ Distance covered by body, $s = \frac{1}{2} a t^2$

$$\because \quad s_{\text{sphere}} = s_{\text{cylinder}}$$
$$\frac{1}{2} a_{\text{sphere}} \times t^2 = \frac{1}{2} \times s_{\text{cylinder}} \times t^2$$
$$\Rightarrow \quad a_{\text{sphere}} = a_{\text{cylinder}}$$
$$\Rightarrow \quad \frac{g \sin 30\Upsilon}{1 + \frac{\frac{2}{5} mR^2}{mR^2}} = \frac{g \sin \theta}{1 + \frac{mR^2}{mR^2}}$$
$$\Rightarrow \quad \frac{1/2}{7/5} = \frac{\sin \theta}{2} \Rightarrow \sin \theta = \frac{5}{7}$$
$$\Rightarrow \quad \theta = \sin^{-1}\left(\frac{5}{7}\right) = 45.6\Upsilon \approx 45\Upsilon$$

Question34

Four spheres each of diameter $2a$ and mass m are placed in a way that their centers lie on the four corners of a square of side b. Moment of inertia of the system about an axis along one of the sides of the square is

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Options:

A. $\frac{8}{5} ma^2$

B. $\frac{4}{5} ma^2 + 5mb^2$

C. $\frac{4}{5} ma^2 + 2mb^2$

D. $\frac{8}{5}ma^2 + 2mb^2$

Answer: D

Solution:

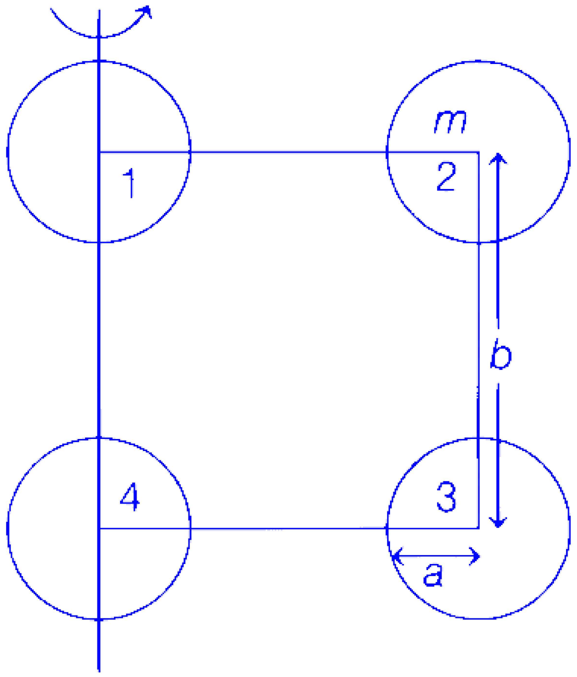
Given,

Diameter of sphere = 2a

Radius of sphere = a

Mass of sphere = m

Side of square = b



$\because$ Moment of inertia of sphere, $I = \frac{2}{5}ma^2$

From parallel axis theorem, $I_p = I_{\text{CM}} + mR^2$

So, net moment of inertia on sphere

$$1 = 2 \times \left(\frac{2}{5}ma^2 + mb^2\right) + 2 \times \left(\frac{2}{5}ma^2\right)$$
$$= \frac{8ma^2}{5} + 2mb^2$$

Question35

If an energy of 684 J is needed to increase thespeed of a flywheel from 180 rpm to360 rpm, then find its moment of inertia.

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Options:

A. 0.7 kg/m^2

B. 1.28 kg/m^2

C. 2.75 kg/m^2

D. 7.28 kg/m^2

Answer: B

Solution:

Given, Energy, $E = 684$ J

Initial angular frequency, $\omega_i = 180$ rpm $= 6\pi$

Final angular frequency, $\omega_f = 360$ rpm $= 12\pi$

Let, I be the moment of inertia.

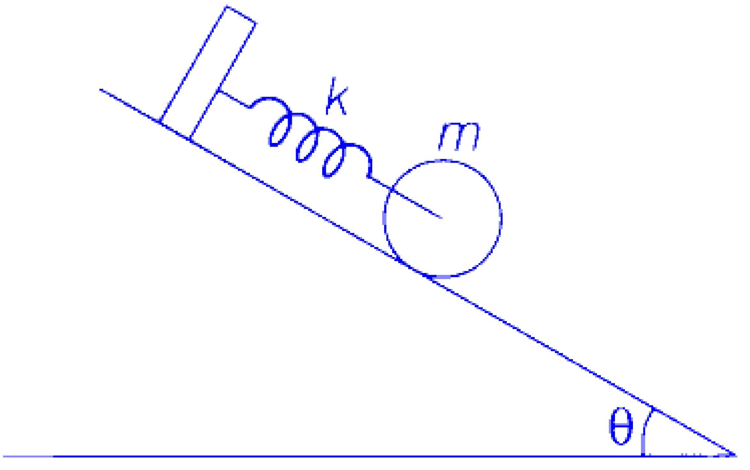
As we know that,

$$E = \frac{1}{2} I \left(\omega_f^2 - \omega_i^2\right)$$
$$684 = \frac{1}{2} I \left[(12\pi)^2 - (6\pi)^2\right]$$
$$\Rightarrow \quad I = \frac{684 \times 2}{\left(144\pi^2 - 36\pi^2\right)} = \frac{684 \times 2}{1065.9} = 1.28 \text{ kg} - \text{m}^2$$

Question36

A sphere of mass m is attached to a spring ofspring constant k and is held in unstretchedposition over an inclined plane as shown inthe

figure. After letting the sphere go, findthe maximum length by which the springextends, given the sphere only rolls.



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Options:

A. $\frac{2mg\sin\theta}{k}$

B. $\frac{k}{2mg\sin\theta}$

C. $\frac{2\sin\theta}{kmg}$

D. $\frac{2mg\cos\theta}{k}$

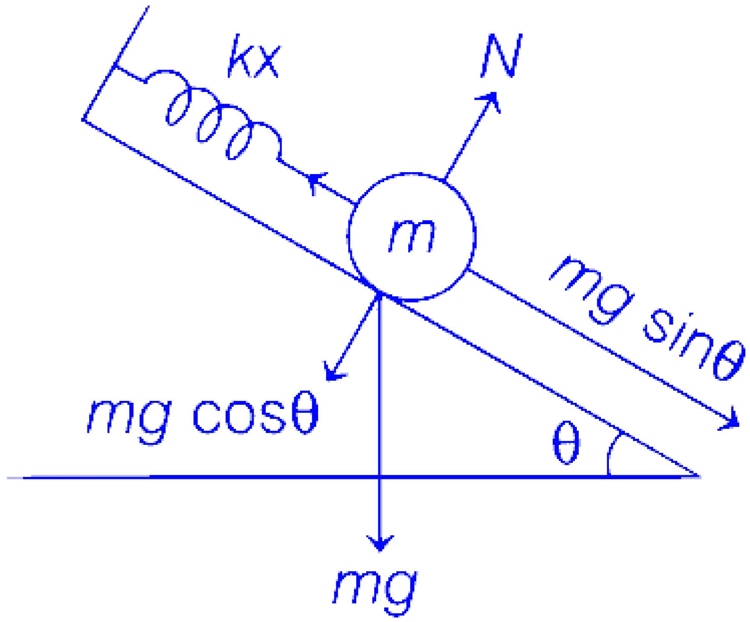
Answer: A

Solution:

Given, mass of sphere $= m$

Spring constant $= k$

Angle of inclined $= \theta$



By using law of conservation of energy,

$$\frac{1}{2}kx^2 = mgx\sin\theta$$
$$\Rightarrow \quad x^2 = \frac{2mgx\sin\theta}{k}$$
$$x = \frac{2mg\sin\theta}{k}$$

Question37

A girl of mass M stands on the rim of africtionless merry-go-round of radius R androtational inertia I, that is not moving. Shethrows a rock of mass m horizontally in adirection that is tangent to the outer edge ofthe merry-go-round. The speed of the rock,relative to the ground is v. Afterwards, thelinear speed of the girl is

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Options:

A. $\frac{mvR^2}{I+MR^2}$

B. $\frac{(m+M)vR^2}{I+MR^2}$

C. $\frac{mvR^2}{I+(M+m)R^2}$

D. $\frac{mvR^2}{I+(M-m)R^2}$

Answer: A

Solution:

Given, mass of girl = M

Radius of marry-go-round = R

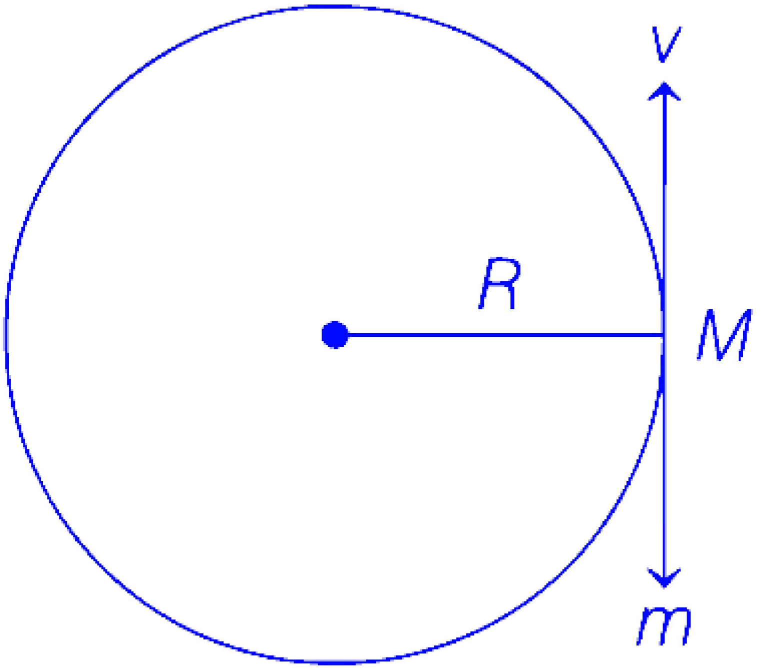
Rotational inertia of girl = I

Mass of rock = m

Speed of rock with respect to ground = v

Let linear speed of girl be v.

By using law of conservation of angular momentum,



Initial angular momentum (L_i) = Final angular momentum L_f

$$\Rightarrow \quad 0 = mvR - \left(I + MR^2\right)\frac{v'}{R}$$

$$\Rightarrow \quad mvR = \left(I + MR^2\right)\frac{v'}{R}$$

$$\Rightarrow \quad mvR^2 = \left(I + MR^2\right)v'$$

$$\therefore \quad v' = \frac{mvR^2}{I+MR^2}$$

Question38

Which of the following type of wheels ofsame mass and radius will have largestmoment of inertia?

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Options:

A. Ring

B. Angular disc

C. Solid disc

D. Cylindrical disc

Answer: A

Solution:

Given, mass M and radius R of wheels are same

As we know that,

Moment of inertia (I) for following bodies are

(i) Ring $= MR^2$

(ii) Angular disc $= \frac{M}{2}\left(R^2 - r^2\right)$

(iii) Solid disc $= MR^2/2$

(iv) Cylindrical disc $= MR^2/2$

Hence, moment of inertia of ring among all four is maximum.
